An exploration and development cycle emphasizing the use of concept maps for subtraction can expand preservice teachers’ understanding.
Mapping the Way to Content Knowledge

By Lisa L. Poling, Tracy Goodson-Espy, Chrystal Dean, Kathleen Lynch-Davis, and Art Quickenton

The role of teacher education is to initiate purposeful conversation regarding specific content, to provide the scaffold for which preservice teachers can begin to understand the subject matter for the purpose of teaching, and to acknowledge gaps in their conceptual knowledge (Grossman and Shoenfeld with Lee 2005). Teacher candidates may enter elementary education programs with basic understanding of whole-number operations but without understanding deeper connections. Building true conceptual understanding requires that individuals learn to make meaningful connections between mathematical concepts. The National Council of Teachers of Mathematics (NCTM) Connections Process Standard emphasizes the need to “recognize and use connections among mathematical ideas; and understand how mathematical ideas interconnect and build on one another to produce a coherent whole” (2000, p. 64). Solidifying connections between concepts and content must be at the forefront of instructing pre-service teacher candidates. A learning tool that we have found irreplaceable to reveal these connections is a visual representation created by the learner: a concept map.
Concept maps
Allowing for the development of a robust understanding, concept maps provide an organized way to show changes in students’ conceptual understanding over time regarding a specific focus topic that implements myriad perspectives and multiple connections (Novak and Cañas 2008). Concept maps are a tool to construct representation of meaning, making connections that are more powerful and thorough than the “rote memorization of facts and procedures” (Grossman and Schoenfeld with Lee 2005, p. 216). The strength of concept maps lies in providing a context in which students can begin to organize personal knowledge and strengthen connections between concepts. The use of concept maps aids in making students aware of what they do not yet know or understand (i.e., concept knowledge gaps) in a nonjudgmental setting and then develop a proactive and positive means for attaining that knowledge.

Choosing a topic
We identify characteristics of a mathematical topic—in this case, subtraction—that we want preservice teachers to master. We determine the evidences needed to establish competency and design a cycle of instructional tasks. The main goals of this instructional cycle are that teacher candidates become aware of their current knowledge of subtraction and gradually become aware of the gaps between their current understanding and the deeper level of understanding that they need so they can teach. We believe that for preservice teachers to become aware of a mathematical concept and their gaps in understanding of that concept, they must encounter it in a variety of contexts that require them to acknowledge and use different representations of the mathematical idea. Therefore, we developed a protocol that requires students to engage with the concept of subtraction in a variety of settings to help them see ideas embedded in subtraction and to recognize how these ideas relate to other mathematical concepts. This instructional cycle focuses on subtraction, but other instructional cycles may be devised to explore other mathematical concepts in depth.

We chose to focus on subtraction for several reasons. First, we wanted to develop a way of thinking about mathematics within the course that could be applied across concepts. This meant that we had to choose the topic early in the term.

Next, we chose subtraction rather than addition because subtraction is generally a more difficult concept for students to learn and for teachers to teach, although preservice teachers tend to think it is an easy topic. They are more confident in their knowledge of this topic as compared to a complicated topic, such as fractions. Thus, because they do not recognize the complexity of the content of subtraction, it is easier to problematize what they think they know about the concept. Definitions of subtraction are examined across whole numbers, rational numbers, and integers. An in-depth knowledge of subtraction requires understanding of equivalent ways to describe the operation of subtraction as well as an understanding of pedagogical elements that influence instruction.

The DCK assignment
To promote the connections and deeper pedagogical and content knowledge that are essential for preservice teachers, we developed the Depth of Content Knowledge (DCK) assignment. Although we did not replicate all parts of the DCK assignment for other math-
ematical topics because of the time required, we were able to refer to parts of this assignment when students were learning other concepts to remind them of ways to go about learning concepts with which they were struggling.

The DCK assignment has four purposeful subsections:

1. Development of a concept map that is iteratively modified after each of the subsequent tasks to yield a final knowledge package
2. Content analysis of student reasoning
3. Content analysis of tasks
4. A content analysis synthesis paper

Each subsection brings attention to aspects of essential mathematical knowledge and will be described below.

The representations used in this article are indicative of typical student submissions. Pairs of preservice teachers complete the concept maps so that they can support and challenge one another’s understandings of the ideas. For the purposes of this article, we focused on one student, Lynley, so that the growth over time would be evident. When making the choice of which student example to reference, we selected this student’s work because of the depth of the individual’s written reflection on growth at the end of the DCK tasks.

Creating a knowledge package

When students struggle to create a concept map representative of their conceptual understanding, they are personally engaging in the development of knowledge. A learner who has spent most of his or her life learning by rote is now required to develop a means for organizing current knowledge along with the assimilation of new information (Novak and Cañas 2008). The first task in the DCK assignment requires students to create a concept map based on their understanding of subtraction without researching any information. Teacher candidates’ initial concept maps (see fig. 1) show that many of them recognize only the procedure of subtraction and have little conceptual knowledge of the connections to, and complexity of, subtraction.

Throughout the DCK assignment, multiple iterations of the concept map are constructed and a final subtraction knowledge package emerges. We refer to the final concept map as a “knowledge package,” following the terminology used by Ma (1999).

Content analysis of student reasoning

The second task that we created to promote the conceptual and pedagogical knowledge of subtraction for our preservice teachers employs a video analysis of student reasoning. The teachers see a video clip of a young child working the subtraction problem 70 – 23. The child, Gretchen, attempts to solve the problem using three different strategies: the traditional algorithm, a base-ten block, and a hundred chart (Philipp and Cabral 2005, IMAP #3-Gretchen).

In the video, Gretchen first solves the problem using the traditional vertical subtraction algorithm:

\[
\begin{array}{c}
70 \\
-23 \\
\hline \\
53 \\
\end{array}
\]
Gretchen explains, “Zero take away three is three.” This misunderstanding impedes her ability to use the standard algorithm.

The interviewer in the video asks Gretchen to try the problem again, this time using base-ten pieces as thinking tools. Gretchen uses the materials and obtains the correct answer, 47. However, Gretchen is convinced that the answer obtained using the algorithm is the correct one. The interviewer asks Gretchen to solve the problem for a third time, using a hundred chart and a marker. Gretchen is again able to successfully solve the problem. She is bewildered by the fact that the answers obtained using the algorithm and the manipulatives do not agree, and she is insistent that the algorithm must be correct. Viewing the clip, preservice teachers can see the discrepancies that exist in the student’s expressed thoughts. They are also able to see the undesirable consequences arising from a student being taught an algorithm before she is conceptually ready to use it accurately. This task, by virtue of the video selected, brought in the idea of using manipulatives to enhance conceptual understanding, and it illustrated how students’ thinking can inhibit their ability to move ahead when instruction is not well sequenced.

This iteration of the concept map shows how Lynley’s depth of knowledge expands on the basis of her experience with the video and class discussions.
We ask the preservice teachers to identify the mathematics embedded in each of the three strategies that the student used, explain the three strategies, describe the inconsistencies among the three strategies, and identify how their mathematical content knowledge has changed. Making sense of subtraction in this task is embedded in the type of work the preservice teachers will do as teachers—making sense of student understanding.

Using the original concept map as a foundation, the teacher candidates begin to construct essential connections. They add the new knowledge acquired from the video and class discussions regarding key elements of subtraction and subtraction solution strategies. Cross-links also begin to appear (see fig. 2). Cross-links, the inclusion of words along the connecting rays of concept maps, describe the relationships between ideas bound by the focus topic. These links create bonds that indicate the personal connections made between domains by the creator of the concept map. Creating cross-links may allow the preservice teacher to see the connections between concepts and ideas that may have seemed independent to them at first glance. This aspect of concept map construction allows deeper knowledge of mathematical content to develop and emerge.

**Content analysis of tasks**

The purpose of the content analysis of mathematical tasks is to develop preservice teachers’ awareness of the cognitive demand of classroom tasks. For this section, preservice teachers assess and organize tasks that pertain to levels of cognitive demand required to complete the task. This activity is intended to draw learners’ attention to important conceptual features, so that they may learn to distinguish between relevant aspects, recognize properties, and/or appreciate relationships between properties.

Based on the work of Stein and her colleagues (2000), content analysis of tasks requires preservice teachers to look at typical classroom tasks and differentiate between high and low cognitive tasks as well as the subcategories found within each. During class, students work with partners to sort a set of place-value tasks (Friel et al. 2008) as low cognitive demand (LCD—e.g., memorization, processes without connections) or as high cognitive demand (HCD—e.g., processes with connections, “doing” mathematics—conjecturing, justifying, or interpreting) tasks (Stein and Smith 1998; Smith and Stein 1998). Then, as a whole class, we discuss why each group sorted the tasks in the way that they did. Following the class discussion, students work with a partner to sort a set of twelve mathematical tasks concerning two-digit subtraction to determine if each task represents an LCD task or an HCD task. Each pair of students notes what makes each task high or low in terms of cognitive demand—including the implications for teaching—by providing evidence of understanding about how LCD and HCD tasks contribute to developing student understanding (see the online appendix). These tasks include a variety of two-digit subtraction tasks intended to help our students become aware of the wide range of learning contexts that this concept can appear within. One of the purposes
behind asking preservice teachers to sort these tasks is to help them realize that different problem representations may elicit different types of student thinking and activity. Some of the problems invite students to model the problem or to examine the thinking and work of an example student. Such tasks are very different from those that ask students to apply an algorithm. One feature of LCD and HCD tasks that the preservice teachers are asked to examine is the possibility of different cognitive entry points to solving the task. HCD tasks typically have multiple entry points. Figure 3 shows subtraction tasks that require students to do the following:

- Write a problem to match a given number sentence (task A)
- Solve a subtraction task written in a missing-addend form (task I)
- Model tasks that are written in the style of Cognitively Guided Instruction (Carpenter et al. 1999) problem types (task B)
- Examine tasks that require them to explain student activity (tasks D and J)
- Perform subtraction by applying algorithms (task E)
- Address tasks requiring them to solve a problem using multiple methods (task C)
- Solve traditional subtraction word problems (tasks F and G)
- Solve subtraction tasks that require students to apply subtraction knowledge to novel situations and to provide justifications for their solution strategies (tasks H, J, and K)
Task L is one of the more interesting tasks. At first glance, it appears to be an HCD task because it shows many different strategies for solving the problem; however, in actuality, it is an LCD task because a child is likely to answer the question, “How would you solve the problem?” by merely choosing one that has been listed, requiring little cognitive effort. As the pairs of preservice teachers discuss these tasks, they must come to grips with the information that each task provides to the student and with the questions that students are to answer. These questions differ in the depth of knowledge that students must exhibit.

Building on the iteration of the concept map created after the Content Analysis of Student Reasoning, preservice teachers now begin to incorporate connections between cognitive demand and to understand how selected tasks can influence a student’s growth (see fig. 3).

The subtraction knowledge package

The last task is to write a content analysis synthesis paper and create a final concept map. The final subtraction knowledge package emerges as a result of iterative changes made to the original concept map after each learning activity.

In the content analysis synthesis paper, the teacher candidates are to show evidence of the development of their personal content knowledge of subtraction using information gained during classroom discussions and activities as well as from required readings of research-based

Lynley’s final concept map and content analysis synthesis paper allow her instructor to see growth in the depth of her understanding of the concept of subtraction and its pedagogy.

**FIGURE 4**
literature and online forum reading responses. In adhering to the idea of scaffolding, this task allows students to bring individual components together, make the necessary connections between them, and solidify their understanding of subtraction. As a capstone task, it gives teacher educators an analysis tool and the means to compare a candidate’s initial map to the final map, depicting the individual’s growth over time and also the depth of his or her understanding of pedagogical and conceptual ideas with regard to subtraction (see fig. 4).

Fortifying instruction

With the culmination of the DCK activity, the final concept map, called the knowledge package, shows a richer understanding of subtraction. Through concept mapping, preservice teachers demonstrate a conceptual as well as procedural understanding of the content and may show how they view connections between ideas. At the completion of this depth-of-content assignment, Lynley stated,

After exploring, analyzing, and working with subtraction, my view of the concept has been significantly altered. I now understand and am prepared to teach the various strategies and methods that exist to perform subtraction problems. I also realize the importance of recognizing and teaching the content knowledge of subtraction before teaching the computation itself. . . . Because I possess this knowledge and plan to act upon it, I hope my students will develop an affinity for mathematics and a desire to explore its relationships and connections both in and out of the classroom. This attitude will shape their future and produce responsible citizens who are able to evaluate, analyze, and solve problems in all areas of their life and in society.

By using concept maps, preservice teachers demonstrate their knowledge and understanding of a concept, linking concepts and joining ideas within math (Afamasaga-Fuata’i 2008). Educators constantly work to find means by which to motivate students to learn mathematics at deepening levels. Although our example of concept mapping is with preservice teachers, the idea of concept mapping to build and fortify instruction reaches far beyond. In a mathematics classroom, concept mapping can be used as not only a learning tool but also a teaching and assessment instrument. Concept maps are notable tools for building connections within a concept and organizing new knowledge for any content area; they are appropriate for any academic level. They afford teachers a window into students’ thinking and can serve as an important formative assessment tool. After each of the carefully sequenced instructional activities, we ask students to revisit their concept map to modify and add to their representation, illustrating their current level of understanding. This iterative process allows preservice teachers to become aware of their mathematical knowledge and how it is changing over time, which empowers them as learners of mathematics.
BIBLIOGRAPHY

Two additional pages of subtraction tasks are appended to this article online.